On Skill Acquisition Support by Analogical Rule Abduction

Koichi Furukawa[1], Keita Kinjo[2], Tomonobu Ozaki[3], and Makoto Haraguchi[4]

[1] Graduate School of Business Innovation, Kaetsu University
kfurukawa@kaetsu.ac.jp
[2] College of Economics and Environmental Policy, Okinawa International University
keita.kinjo@okiu.ac.jp
[3] College of Humanities and Sciences, Nihon University
tozaki@chs.nihon-u.ac.jp
[4] Graduate School of Information Science and Technology, Hokkaido University
makoto@kb.ist.hokudai.ac.jp

Abstract. This paper describes our development of analogical abduction as an extension to our work on meta level abductive reasoning for rule abduction and predicate invention. Previously, we gave a set of axioms to state the object level causalities in terms of first-order-logic (FOL) clauses, which represent direct and indirect causalities with transitive rules. Here we extend our formalism of the meta level abductive reasoning, by adding rules to conduct analogical inference. We have applied our analogical abduction method to the problem of explaining the difficult cello playing techniques of spiccato and rapid cross strings of the bow movement. Our method has constructed persuasive analogical explanations about how to play them. We have used a model of forced vibration mechanics as the base world for spiccato, and the specification of the skeletal structure of the hand as the basis for the cross string bowing technique. We also applied analogical abduction to show the effectiveness of a metaphorical expression of "eating pancake on the sly" to achieve forte-piano dynamics, and successfully created an analogical explanation of how it works.

Keywords: rule abduction, analogical abduction, predicate invention, predicate identification, cello playing

1 Introduction

Abduction is a kind of synthetic reasoning to construct explanatory hypotheses about surprising observations. Here we explain how we have succeeded in applying abductive inference to provide explanations about how to perform difficult cello playing techniques, by exposing previously "hidden secrets" behind what are sometimes called a "knack" for a particular technique.

Knacks play crucial roles in acquiring artistic or sports skills. Knacks are target-dependent and are expressed by such phrases as "if you want to achieve a target exercise A, you should do an action B". But typically it is difficult to explain why the action B works for achieving the exercise A because of the "hidden secrets" behind the knack.

This problem setting fits the abduction framework quite well. A knack is usually a surprising observation and therefore hypotheses generation by abduction can help in finding candidates for the "secret" prerequisite for achieving the given exercise. To elaborate, we try to abduce missing hypotheses to achieve the goal (exercise) A under the assertion of the fact (action) B. Since B appears at the leaf of the proof tree, the abduction procedure has to find hypotheses in between the goal A and the leaf B, identified as a (set of) rule(s). We refer to this abductive procedure as *rule abduction*. Rule abduction cannot be achieved by standard Abductive Logic Programming (ALP), because abducibles are limited only to facts in ALP. To solve the difficulty, we developed a rule abduction method using *meta level* abduction [1, 2].

However, our rule abduction alone is insufficient to obtain meaningful missing prerequisites in the real application domain of skill acquisition. For example, consider an example of a knack "you should bend the thumb joint to realize crossing strings quickly." In this example, a missing rule is the knack itself; that is, "to achieve crossing strings quickly, bend the thumb joint" is a rule to be hypothesized by rule abduction. But it is easy to see that this rule is useless, because it does not explain *why* it works effectively.

Here we introduce an analogical abduction system which makes it possible to give a suitable explanation to the proposed knack. To show the effectiveness of the knack, we need to identify a hidden reason. The hidden reason is typically provided by analogical reasoning which gives a possible explanation of the knack by means of an argument in an underlying analogical domain associated with the original vocabulary of the abducible rules. There may be a situation where a (set of) intermediate proposition(s) is necessary to fill a gap between the premise B of the knack and its goal A, in which case we need to invent a new node (predicate) betwen them.

Note that some studies, [3, 4], discuss the relationship between analogy and abduction (or induction) to complete missing clauses. Defourneaux et al. [3] proposed to use abduction to hypothetica lly assume some clauses with which proofs in a base domain can be transformed into those in a target one. A major part of target domain for deduction is thus obtained by analogy, while some auxiliary clauses are supplemented by abduction to complete a proof in the target domain. In contrast with such a usage of abduction, abduction in this paper is to build similarity (correspondence) between graph vertices according to which causality (represented by graph edges) is transformed and guessed in the target domain.

A similar construction of target clauses can be also realized by inductive inference using positive and negative examples. A learning system presented in [4] calculates a major part of target program by applying hypothetical analogy mapping to a given set of program clauses of a base domain. After the mapping, a refinement operator of inductive inference is applied to the major part in order to have the target clauses so that the reasoned program clauses in the target domain are consistent with the examples. Neither induction nor abduction are not directly used to hypothetically assume analogy correspondence, similar to the case of [3].

There is another research which combines abduction and case-based reasoning[5]. They have tried to incorporate case-based reasoning into abductive logic programming and have succeeded in automatically finding pairs of similar objects by abduction. On the other hand, we focus analogous systems having causality where we try to identify corresponding objects having similar roles in their causality relations.

We consider three analogical abduction problems and propose possible procedures to give solutions. In Section 2, we give a summary of our previous work on rule abduction and predicate invention. In Section 3, we incorporate analogical inference into rule abduction. In Section 4, we show programs for the three concrete examples and their solutions. Finally in Section 5, we discuss several issues of our approach and give concluding remarks.

2 Rule Abduction and Predicate Invention

In this section, we give the summary of our previous research to realize rule abduction and predicate invention [1, 2].

Definition 1. *Let B be a set of clauses representing background knowledge, and G a set of literals representing observed events. Consider a set Γ of literals that can be assumed to be true. Each member of Γ and any instance of an element of Γ is called an abducible literal, and the predicate of an abducible literal is called an abducible predicate.*

Given B, G, and Γ, abductive reasoning infers a set H of abducible literals such that

$$B \cup H \models G \tag{1}$$

$$B \cup H \text{ is consistent, and} \tag{2}$$

$$H \text{ is a set of instances of literals from } \Gamma \tag{3}$$

Then, H is called an explanation of G (with respect to B and Γ).

Each literal in H can contain variables, which are assumed to be existentially quantified. If H does not contain any variables, it is called a ground explanation.

In Definition1, the first condition states that the observed event G can be explained by augmenting B with an additional hypothesis H. Because we use clausal theories instead of logic programs, an integrity constraint is represented as a negative clause in B, and the second condition in Definition1 corresponds to the satisfaction of integrity constraints.

SOL (Skip Ordered Linear) resolution [6] is a calculus that realizes abductive reasoning in full clausal theories, and SOLAR [7] is a tableaux-based implementation of SOL resolution.

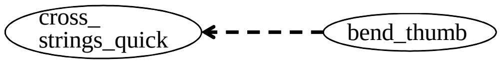

Fig. 1. An empirical causality representing that bending the thumb causes quick strings crossing

To implement abductive reasoning with SOL resolution, we have to convert the first condition in Definition1 into the following formula. This is an application of the relation known as inverse entailment.

$$B \cup \neg G \models \neg H \tag{4}$$

Since, both G and H can be regarded as conjunctions of literals, both $\neg G$ and $\neg H$ are clauses. On the other hand, the second condition in Definition1 is equivalent to $B \not\models \neg H$. Hence, to compute an explanation of G in abductive reasoning, a theorem of B and $\neg G$ which is not a theorem of B is deduced as $\neg H$, which is then negated as H. In this case, since any element of H is an abducible literal, any literal in $\neg H$ is the negation of an instance of an element of Γ. Moreover, since any theorem $\neg H$ deduced from a clausal theory in SOL resolution is computed as a clause, every variable contained in it is universally quantified, and is thus existentially quantified in its negation H.

Suppose that we empirically know a cause s brings a remarkable result g. Here, s and g are called a *cause event* and a *result event*, respectively. This relation is actually an empirical causality whose example is given by Fig.1. Our task of rule abduction is to explain why or how this causality holds, by finding an explanation that fills the gap between a causal event and a causal result.

A *causal graph* is a directed graph representing causal relations, and consists of a set of nodes and arcs. A *direct causal relation* corresponds to an arc, and a *causal chain* is represented by the reachability between two nodes.

When there is a direct causal relation from the node s to the node g, we declare that $connected(g, s)$ is true, shown by an atom (5). If we know that there is no direct causal link from s to g, we add an *integrity constraint* of the form (6).

$$connected(g, s) \tag{5}$$

$$\neg connected(g, s) \tag{6}$$

When there is a *causal chain* from s to g, we declare that $caused(g, s)$ is true. We then have the following formulas as axioms:

$$caused(X, Y) \leftarrow connected(X, Y) \tag{7}$$

$$caused(X, Y) \leftarrow connected(X, Z) \wedge caused(Z, Y) \tag{8}$$

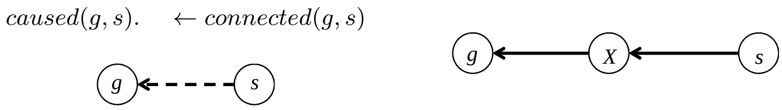

Fig. 2. An observed indirect causal relation and its clausal form representation (left), and a causal graph corresponding to a hypothesis with a new predicate (right).

Here, the predicates *connected* and *caused* are both meta-predicates for object-level propositions g and s. From now on, we refer to this representation of causality relations as Meta Level Causality (MLC) representation.

Rules, like causal relations at the object level, are represented by atoms in the meta level. In this way, we can implement *rule abduction* in the object level as *fact abduction* in the meta level.

When a causal graph has defect, there is no path between a goal event g and an input event s. Now an abductive task can be used to infer missing links (and sometimes missing nodes) to complete a path between the two nodes. This is done by setting the abducibles Γ as atoms containing connected only: $\Gamma = \{connected(\_, \_)\}$. The observation is given in the form of the causal chain $caused(s, g)$, but we usually assume that there is no direct causal relation between them, i.e., $\neg connected(g, s)$, otherwise we would not have needed abduction.

Suppose an observation $caused(g, s)$ is given together with a constraint $\neg connected(g, s)$. The clausal form of the observation and the constraint, and their corresponding causal graph are given in Fig.2(left):

Within this specification, a possible explanation has the following form:

$$\exists X \; (\, connected(g, X) \wedge connected(X, s) \,) \tag{9}$$

This X can be unified with some known node in the causal graph, but if it is assumed as a new node, this assumption is equivalent to predicate invention [8]. See Fig. 2(right). Note here that, to introduce these kinds of explanations, we need to allow *existentially quantified formulas* as hypotheses. Abduction by SOLAR enables us to infer hypotheses having this form as stated above.

3 Analogical Abduction

In this section, we incorporate analogical reasoning into our MLC framework. We refer to the world under consideration as the target world and the corresponding analogical world as the base world. Analogical reasoning is achieved by introducing a base world similar to the target world, where we conduct inference [9]. Analogical reasoning can be formulated as logical inference with equality hypotheses [10]. We achieve analogical abduction by extending our MLC based rule abduction framework.

We modify the causality relationship formula (7) and (8) to deal with causalities in the different worlds separately as follows:

$$t\_caused(X,Y) \leftarrow t\_connected(X,Y) \tag{10}$$
$$t\_caused(X,Y) \leftarrow t\_connected(X,Z) \wedge t\_caused(Z,Y) \tag{11}$$
$$b\_caused(X,Y) \leftarrow b\_connected(X,Y) \tag{12}$$
$$b\_caused(X,Y) \leftarrow b\_connected(X,Z) \wedge b\_caused(Z,Y) \tag{13}$$

where the prefix "*t_*" represents a predicate in the target world and "*b_*" in the base world. Although the predicate "*b_caused*" does not appear in following examples, we define it because of the symmetry with "*t_caused*," for the possible future use.

We also introduce a predicate "$similar(X,Y)$" to represent similarity relations between an atom X in the target world and a corresponding atom Y in the base world.

Now we have to define the predicate "*t_connected*," for which we have to consider three cases to show the connectedness in the target world; the first case is that the connectedness holds from the beginning, (14); the second case is that it holds by abduction as a solution of abductive inference, (15); and the third case is that it is derived by analogy, (16). Definition (16) contains an auxiliary predicate "$print\_connected\_by\_analogy(X,Y)$" which indicates that it is to be "*printed*" as a part of an abduced hypothesis to provide evidence that the analogical connection is actually used to show the "*t_connected*"ness. Since analogical reasoning can be achieved without any defects in the inference path, we need to prepare an artificial defect, "$print\_connected\_by\_analogy(X,Y)$", on the path. This printing in turn is defined by specifying the predicate "*print_connected_by_analogy*" as an abducible.

$$t\_connected(X,Y) \leftarrow connected\_originally(X,Y) \tag{14}$$
$$t\_connected(X,Y) \leftarrow connected\_by\_abduction(X,Y) \tag{15}$$
$$t\_connected(X,Y) \leftarrow connected\_by\_analogy(X,Y) \wedge print\_connected\_by\_analogy(X,Y) \tag{16}$$

We have to further define three predicates; "*connected_originally*", "*connected_by_abduction*" and "*connected_by_analogy*". The predicate "*connected_originally*" is used in the assertion of facts representing the original connection; "*connected_by_abduction*" is introduced as an abducible predicate. Finally, the definition of "*connected_by_analogy*" is given by the following analogy axiom which plays a central role in analogical abduction.

Analogy Axiom

$$connected\_by\_analogy(X,Y) \leftarrow b\_connected(XX,YY) \wedge similar(X,XX) \wedge similar(Y,YY) \tag{17}$$

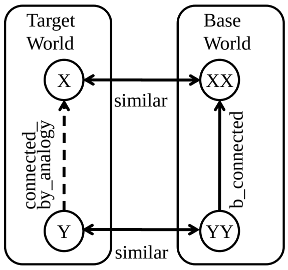

Fig. 3. A scheme representing the Analogical Axiom.

This axiom states that the nodes X and Y in the target world can be linked by the predicate "$connected\_by\_analogy(X, Y)$" because of the base relationship "$b\_connected(XX, YY)$" between XX and YY which are similar to X and Y, respectively, as shown in Fig. 3. Note that there may be more than one similarity candidates. In this paper, we assume that the user provides some of the initial similarities, and that the abductive inference engine will compute any remaining possible similarity hypotheses to explain the observation.

Finally we state an important integrity constraint that *connected_by_abduction* and *connected_by_analogy* does not hold simultaneously as expressed as follows:

$$\leftarrow connected\_by\_analogy(X, Y) \wedge connected\_by\_abduction(X, Y) \quad (18)$$

4 Analogical Abduction Examples

In this section, we show three analogical abduction examples in cello playing domain. The first one is a problem of discovering similarities to establish analogy between two given worlds. The second one is to conduct both predicate invention and similarity discovery at the same time. The third one is applying analogical abduction to metaphorical expression.

4.1 Discovering similarities

We consider a problem of achieving the cello playing technique called spiccato, by analogy with forced vibration. Because of a cello instructor's suggestion, we happened to know that "holding the bow by the ring finger" is an essential action to achieve spiccato. In addition, from our intuition about the physics of such skills, we believe that the forced vibration is achieved by both "keeping the timing of energy supplying just after the maximum amplitude" and "absorbing shock of the energy supply." The similarity to be discovered here is the one between "holding the bow by the ring finger" and "absorbing shock of the energy supply." This similarity suggests that spiccato is achieved by holding the bow by the ring finger in order to absorb shock of hitting the bow to a string (to supply energy to the bow to bounce continuously). The relationship of the target world of achieving spiccato and the base world of forced vibration is shown in Fig. 4.

The analogical abduction program is given as follow. We use Prolog-like notation here for the readability. In the program, the notation "Pred/N", such as connected_by_abduction/2, denotes a predicate "Pred" having "N" Arities.

```
% Observation (G) :
t_caused(spiccato, support_bow_with_ringfinger).                          (19)
% Abducible predicates(Γ) :
abducibles([connected_by_abduction/2, similar/2,
                              print_connected_by_analogy/2]).
% Background Knowledge(B) :
%%% Base world:
b_connected(forced_vibration, shock_absorber).                            (20)
%%% Target world:
:-connected_by_abduction(spiccato, support_bow_with_ringfinger).          (21)
% Similarity:
similar(spiccato, forced_vibration).                                      (22)
%Axioms:
b_caused(X, Y):-b_connected(X, Y).
b_caused(X, Y):-b_connected(X, Z), b_caused(Z, Y).
t_caused(X, Y):-t_connected(X, Y).
t_caused(X, Y):-t_connected(X, Z), t_caused(Z, Y).
t_connected(X, Y):-originally_connected(X, Y).
t_connected(X, Y):-connected_by_abduction(X, Y).
t_connected(X, Y):-connected_by_analogy(X, Y), print_connected_by_analogy(X, Y).
connected_by_analogy(X, Y):-b_connected(XX, YY), similar(X, XX), similar(Y, YY).
```

In this program, the goal (observation) to be satisfied is "`t_caused( spiccato, support_bow_with_ringfinger )`" (clause (19)). We provide the following two facts: 1) "`shock_absorber`" is one of the possible causes to achieve the forced vibration (clause (20)), and 2) spiccato is analogous to the forced vibration (clause(22)). In addition, we provide a negative clause asserting that the direct connection from "`support_bow_with_ringfinger`" to "`spiccato`" cannot be hypothesized (clause (21)).

In a SOLAR experiment, the number of obtained hypotheses is 7 when the maximum search depth is set to 10 and the maximum length of produced clauses is 4. One plausible hypothesis is:

`print_connected_by_analogy(spiccato, support_bow_with_ringfinger)` $\wedge$

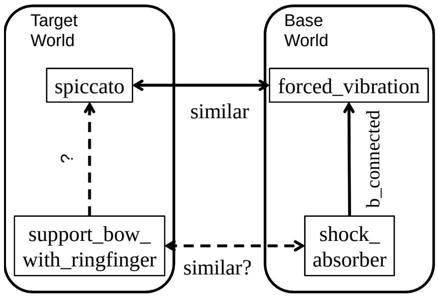

Fig. 4. Analogical abduction for achieving spiccato playing. The set of dotted lines are to be computed as a hypothesis.

$$\texttt{similar}(\texttt{support\_bow\_with\_ringfinger}, \texttt{shock\_absorber})$$

which indicates that the support of the bow with the ring finger in achieving spiccato is analogous to the shock absorber in the forced vibration as shown in Fig.4. Note that we gave a similarity between spiccato and forced_vibration and obtained another similarity between support_bow_with_ringfinger and shock_absorber by analogical abduction. It is note worthy to mention that we need not provide any definition of the "similar" predicate in abducing similarity predicates since they can be automaticaly obtained by the abduction engine which tries to fill gaps to make the proof of causality relation 19 complete.

4.2 Analogical Abduction with Predicate Invention

In this subsection, we consider the problem of showing the effectiveness of bending the thumb to realize the quick crossing of strings (`cross_strings_quick`). We use the skeletal structural linkage of the knuckle (of the first four fingers) and the thumb (`b_connected(knuckle, thumb)`) as a counterpart of a functional linkage of bending the knuckle and bending the thumb (`t_connected(knuckle_bend, thumb_bend)`) in the analogy setting. Note that we define the similarity only between "`bending_thumb`" and "`thumb`" without providing the predicate "`bend_knuckle`", which is to be invented by abductive reasoning. In this example, we conduct discovering missing similarities and invent a predicate at the same time. The problem structure is shown in Fig.5.

The abduction program for this problem is shown as follows (axiom clauses are omitted here as well):

% Observation($\texttt{G}$) :

$\texttt{t\_caused}(\texttt{cross\_strings\_quick}, \texttt{bend\_thumb})$.

% Abducible predicates(Γ) :

$\texttt{abducibles}([\texttt{connected\_by\_abduction}/2, \texttt{similar}/2,$

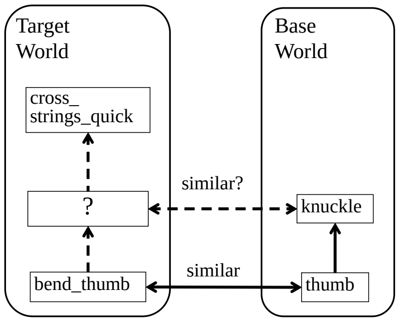

Fig. 5. Analogical abduction with predicate invention.

```
                                         print_connected_by_analogy/2]).
% Background Knowledge(B) :
%%% Base world:
b_connected(knuckle, thumb).
%%% Target world:
:-connected_by_abduction(cross_strings_quick, bend_thumb).
% Similarity:
similar(bend_thumb, thumb).
```

Under the same condition as before, we obtained 7 hypotheses, one of which is the following:

$$\texttt{connected\_by\_abduction}(\texttt{cross\_strings\_quick}, \_0) \wedge \\ \texttt{similar}(\_0, \texttt{knuckle}) \wedge \\ \texttt{print\_connected\_by\_analogy}(\_0, \texttt{bend\_thumb})$$

This hypothesis accurately represents the structure shown in Fig. 5. We further conducted our experimental study by deleting the similarity relation "`similar(bend_thumb, thumb)`" from the above program and then succeeded in recovering this link as well.

4.3 Explaining the Effectiveness of Metaphorical Expression

To show the applicability of our approach to different kinds of problems other than mechanical models, we apply our analogical abduction to explain the effectiveness of a metaphorical expression. An example of metaphorical expression, issued by a trainer to achieve forte-piano dynamics in orchestra rehearsal, is "eating pancake on the sly," which means that one takes a big mouthful of pancake first, and then he/she tries to make it secret by a motion of imperceptible action of chewing. The difficulty of achieving such dynamics arises because we

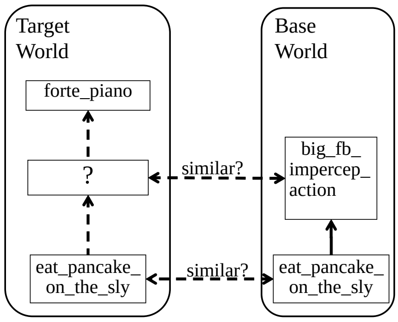

Fig. 6. Mataphorical expression of "eating pancake on the sly" to achieve forte-piano.

cannot control our muscle strength because of an inability to precisely estimate force. In addition, it is quite difficult to attain consensus amongst players about the shape of the dynamics envelope. But a metaphorical expression can sometimes help achieve a consensus. This phenomenon is formalized in terms of our analogical abduction framework.

Our goal is to prove "caused_by(forte_piano, eat_pancake_on_the_sly)". We assume that the expression "eating pancake on the sly" induces a sequence of motor control commands indicating a big action followed by an imperceptible action ("big_fb_impercep_action") in the brain, which arises within the metaphorical base world (see Fig. 6). The analogical abductive reasoning is shown as follows:

```
% Observation(G) :
t_caused(forte_piano, eat_pancake_on_the_sly).
% Abducible predicates(Γ) :
abducibles([connected_by_abduction/2, similar/2,
                              print_connected_by_analogy/2]).
% Background Knowledge(B) :
%%% Base world:
b_connected(big_fb_impercep_action, eat_pancake_on_the_sly).
%%% Target world:
:-connected_by_abduction(forte_piano, eat_pancake_on_the_sly).
```

Under the same condition as before, we obtained 6 hypotheses, one of which is the following:

$$\begin{gathered}\texttt{connected\_by\_abduction}(\texttt{forte\_piano}, \_0) \wedge \\ \texttt{similar}(\_0, \texttt{big\_fb\_impercep\_action}) \wedge \\ \texttt{similar}(\texttt{eat\_pancake\_on\_the\_sly}, \texttt{eat\_pancake\_on\_the\_sly}) \wedge \\ \texttt{print\_connected\_by\_analogy}(\_0, \texttt{eat\_pancake\_on\_the\_sly})\end{gathered}$$

Note that the entire problem structure of this analogical abduction is almost the same as our previous predicate invention example shown in Fig.5 except for the treatment of the similarity relation on the bottom; it is abduced in the metaphorical analogy case whereas it is given from the beginning in Fig.5. The characteristics of the metaphorical analogy is that the same analogical expression appears in both the base and the target worlds. Since a metaphorical expression directly induces emotional feeling to produce adequate motor control commands for achieving the given goal, it should be included in the target world; on the other hand, the same metaphorical expression triggers a similar motion in the eating action which means that it should be in the base world. Another remark is that the metaphorical expression of "eating pancake on the sly" plays the role of converting a quantitative direction of the sound volume adjustment into a qualitative one, which is much more intuitive and understandable to human.

5 Discussion and Future work

In this paper, we proposed an approach of combining rule abduction and analogical reasoning by adding analogy axioms to the original causality relation axioms. We introduced the framework of defining the target world and the base world for the analogical setting as a part of the added analogy axioms.

We succeeded in demonstrating our new analogical abduction engine by applying it to three cello exercises to obtain analogical explanations: one is the similarity discovering problem between a predicate "holding the bow with ring finger" in the target world, and a predicate "absorbing shock" in the base world, as shown in Fig.4; the second example uses both predicate invention and analogical reasoning to explain the invented predicate as shown Fig.5, which is our original goal of this paper; and the final example shows the effectiveness of using metaphorical expression to achieve proper dynamics of forte-piano, as shown in Fig.6. The last example shows the richness of our approach, which covers not only mechanical theories as the base world but also metaphorical expressions which inspire our brain.

Although we intended to incorporate analogical inference into abduction, we failed to achieve the introduction of "structural analogy," since our formulation considers substituting only single connectedness in the target world by the corresponding one in the base world. One possible realization of the structural analogy is to "import" parts of the proof tree in the base world inference into the target world. The essential problem is to introduce "and" connection of the proof tree in the base world into the target world's proof. For example, in our spiccato domain, the forced vibration is achieved by "keeping the timing of energy supplying just after the maximum amplitude" and "absorbing shock of the energy supply." Our attempt was to conduct analogical reasoning by focusing only the second condition of requiring a shock absorber. We will need a more elaborate formulation to deal with this problem.

There is another fundamental issue to be addressed to achieve more realistic analogical abduction. In this paper, we explicitly provide a base world analogous to the target world. In real problems for discovering or explaining skills, we may need to find an appropriate base world itself, before being able to conduct analogical reasoning, or to find and extract similar sub-worlds adequate for analogical abduction from the given target and base worlds. To deal with these problems, we have to provide detailed attributes to the components of each world and compute the degree of similarity for each pair of subset to find analogous pairs [11].

In this paper, we investigated possible ways to incorporate analogical reasoning with a well established ALP system SOLAR. In our approach, we put an abduction engine in the center and tried to add analogical reasoning on it. However, there are other possibilities to generalize our approach further to find better integration of abduction and analogy, including metaphor. One viewpoint is to make analogical reasoning propose adequate abducibles for abduction. This holds promise for strengthening the capability of abductive reasoning by adding the feature of automatic preparation of abducibles. Another view point is to use abduction to propose appropriate similarity relations to establish analogical reasoning, which has been reported in this paper. In other words, abduction and analogy are supporting for each other. An ideal implementation of a complementary abduction-analogy system is future research work.

Acknowledgement

We express our special thanks to Professor Katsumi Inoue from National Institute of Informatics, Professor Hidetomo Nabeshima from Yamanashi University and Professor Randy Goebel from Alberta University for their suggestions and fruitful discussions on how to incorporate analogy into abduction. This research was conducted in the Grants-in-Aid for Scientific Research Category C: Supports on Skill Discovery by Rule Abduction and Analogy, Grant No. 24500183 .

References

1. Furukawa, K., Inoue, K., Kobayashi, I., and Suwa, M.: Discovering Knack by Abductive Reasoning, 23rd Congress of JSAI, 1K1-OS8-5(2009) (in Japanese)
2. Inoue, K., Furukawa, K., Kobayashi, I., and Nabeshima, H.: Discovering Rules by Meta-level Abduction, *Proc.* 19th International Conference on Inductive Logic Programming (ILP 2009), pp.49-64(2009)
3. Defourneaux, G. and Peltier, N.: Analogy and Abduction in Automated Deduction, *Proc.* 15th International Joint Conference on Artificial Intelligence (IJCAI-97), Vol.1, pp.216–225(1997)
4. Sadohara, K. and Haraguchi, M: Analogical Logic Program Synthesis from Examples, *Proc.* 8th European Conference on Machine Learning (ECML-95), LNCS, Springer, Vol.912, pp.232–244(1995).
5. Satoh, K.: Translating Case-Based Reasoning into Abductive Logic Programming, *Proc.* 12th European Conference on Artificial Intelligence (ECAI 1996), pp.142–146(1996)

6. Inoue, K.: Linear Resolution for Consequence Finding, *Artificial Intelligence*, Elsevier, Vol.56, No.2/3, pp. 301-353(1992)
7. Nabeshima, H., Iwanuma, K., and Inoue, K.: SOLAR: A Consequence Finding System for Advanced Reasoning, *Proc.* International Conference on Automated Reasoning with Analytic Tableaux and Related Methods (TABLEAUX 2003), LNCS, Springer, Vol.2796, pp.257-263(2003)
8. Muggleton, S. and Raedt, L.D.: Inductive Logic Programming: Theory and Methods, *The Journal of Logic Programming*, Elsevier, Vol.19/20, pp.629-679(1994)
9. Haraguchi, M. and Arikawa, S.: A Formulation of Analogical Reasoning and Its Realization, *Journal of Japanese Society for Artificial Intelligence*, Vol.1, No.1, pp.132-139(1986) (in Japanese)
10. Goebel, R.: A Sketch of Analogy as Reasoning with Equality Hypotheses, *Analogical and Inductive Inference*, LNCS, Springer, Vol.397, pp.243-253(1989)
11. Haraguchi, M.: Towards a Mathematical Theory of Analogy, *Bull. Informatics and Cybernetics*, Vol.21, No. 3/4, pp.29-56(1985)